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A2_6 Another Zombie Epidemic

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Abstract

We investigate the SIR model applied to a zombie epidemic and introduce new parameters such as the rate at which zombies are killed. We find that adding such parameters makes it far more feasible for a given simulated population to survive a zombie epidemic.

Introduction

In our previous paper 'A Zombie Epidemic' we developed our take on the SIR model, coined the SZD model, and applied it to the spread of a zombie virus. The model allows us to see how the human and zombie populations change with time in a zombie epidemic. We found previously that the population would be unable to survive much past 100 days [1]. In this paper we build upon our SZD model by allowing the infection probability β to vary as the system evolves, accounting for survivors being able to kill the zombies and considering the population's reproduction rate.

Developing the SZD model

As it stands the SZD model is governed by the following set of differential equations

$$\frac{dS}{dt} = -\frac{\beta SZ}{N} \tag{1}$$

$$\frac{dZ}{dt} = \frac{\beta SZ}{N} - \gamma Z \tag{2}$$

$$\frac{dD}{dt} = \gamma Z \tag{3}$$

Where S is the population that is susceptible to the zombie virus, Z is the number of zombies, D is the number of people that have died, β is the probability of becoming infected each day, γ is the reciprocal of the lifetime of a zombie (its time constant) and N is the total population at $t = 0$, (7.5×10^9) [2].

Here, the probability of infection upon an encounter β is constant. However, as more of the susceptible population becomes infected, the survivors are less likely to become infected since they will have experience at avoiding the zombies. We will redefine β as given by Eq (4) so that the infection probability decreases with a decreasing susceptible population.

$$\beta = \alpha S \tag{4}$$

Where α is a normalisation constant such that at $S = 7.5 \times 10^9$ (day 0) the infection probability is 0.9, meaning $\alpha = 1.2 \times 10^{-10}$. By considering the redefined β, Eq's (1) and (2) become

$$\frac{dS}{dt} = -\frac{\alpha S^2 Z}{N} \tag{5}$$

$$\frac{dZ}{dt} = \frac{\alpha S^2 Z}{N} - \gamma Z \tag{6}$$

Additionally we have extended the lifetime of a zombie (from 20 days as previously set) to 1 year to make the zombies more formidable. This implies $\gamma = 1/365$.

Next we alter the model to allow members of the susceptible population to kill the zombies. Killing zombies will decrease the rate of change of the zombie population by the rate at which the susceptible population are able to kill the zombies. This will increase the rate of change of the dead population by the same amount. If each member of the susceptible population has a probability κ of killing a zombie each day, then the rate at which zombies are killed will be given by Eq (7)

$$K = \frac{\kappa S Z}{N} \tag{7}$$

Since κ is the probability of a kill each day, S/N is the fraction of the population that are able to kill a zombie and Z is the number of zombies, the product of these three parameters gives the rate at which zombies are killed. For the purposes of this model we have $\kappa = 0.1$. Therefore K must be subtracted from Eq (6) and added to Eq (3). This gives

$$\frac{dZ}{dt} = \frac{\alpha S^2 Z}{N} - \gamma Z - \frac{\kappa S Z}{N} \tag{8}$$

$$\frac{dD}{dt} = \gamma Z + \frac{\kappa S Z}{N} \tag{9}$$

Finally we consider the rate at which the population will be able to grow through reproduction. Here we assume that at any given time half of the population are able to pair up and reproduce, giving birth to 1 baby every 3 years. This means that the susceptible population will grow at a rate

$$\frac{dS}{dt} = \frac{(1/4)S}{3 \times 365} \tag{10}$$

This model is actually unstable for large populations, but it is suitable as long as the population is small (as the solution is a boundless exponential increase). The factor of 1/4 comes from our assumption that half of the population are female and half of the females are able to have a child. This is then reduced by a factor of 3×365 since the increment dt is in units of days and children are born at a rate of once every 3 years per female. Eq (10) must be added to Eq (5) in order to complete the model. This is given by Eq (11)

$$\frac{dS}{dt} = \frac{(1/4)S}{3 \times 365} - \frac{\alpha S^2 Z}{N} \tag{11}$$

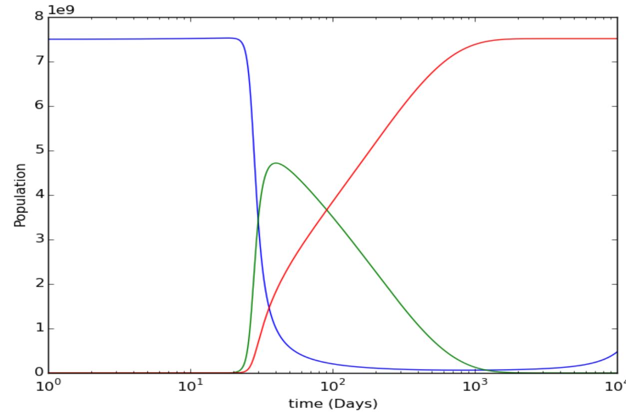

Figure 1: The time evolution of the populations in our SZD model. The blue curve is S, the green curve is Z and the red curve is D.

Discussion and Conclusions

With the aforementioned additions, the amended SZD model is now governed by Eq's (11), (8) and (9). With initial conditions at $t = 0$ as $S(0) = 7.5 \times 10^9$, $Z(0) = 1$, $D(0) = 0$ we find the population evolves in a manner shown by Fig (1). Interestingly we find that it is actually possible for our population to survive the zombie epidemic under these conditions, it can also be seen that once the zombie population has been wiped out at roughly 10^3 days the population starts to recover at 10^4 days.

References

[1] C.T. Davies et al. A2_5 A Zombie Epidemic PST. Vol 15. 2016

[2] `https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology` accessed on 25/10/16