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P5_3 The Moon - Earth Balance

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Abstract

Investigated in this paper is what would happen to the height of the tides if the Moon were closer to the Earth. We found that as the distance between the Earth and Moon decreases, the tidal height increases drastically; a consequence of these tides could be global floods.

Introduction:

Tides are the result of the effects that the Sun and Moon have on Earth. If the Earth were only spinning on its own axis the oceans on Earth would be in equilibrium, this is because the forces acting on it would be the gravitation force acting inwards which is equal to the centrifugal forces acting outward. However, due to the Moon and the Sun, the Earth will have external forces acting on it. The Moons gravitational force exerted on the Earth creates a tidal force, this pulls the water towards the Moon causing the water to bulge (see figure 1). As seen in figure 1 the water also bulges outwards on the side of the Earth facing away from the Moon this is caused by the centrifugal force because at that point is it greater than the Moon's gravitational force.

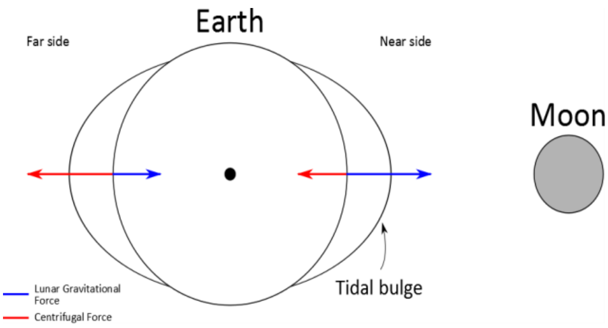

Figure 1 – Schematic of the Earth-Moon system and the forces acting on the tide.

The effect of the Sun's tidal forces is much smaller than the Moon's, this is because the Sun is further away from the Earth. For simplicity we have decided to ignore the Sun's effects on the tides. The height of tides depending on only the change in distance of the Moon away from the Earth will be calculated.

Discussion

To begin with we had to determine how close we could get the Moon to the Earth without catastrophic damage to the Earth. To do this we had to calculate the Roche limit of the two bodies; the Roche limit is the distance in which the tidal force of a body becomes greater than the self-gravitational force of the satellite, at this distance the satellite will be ripped apart. The Roche limit equation [1]:

$$R_{Roche} = \left(\frac{2M}{m}\right)^{\frac{1}{3}} R_m \qquad (1)$$

where R_{Roche} is the Roche limit, M is the mass of the Earth, m is the mass of the Moon and R_m is the radius of the Moon. We calculated R_{Roche} to be 9480km from the centre of the Earth to the centre of the Moon. From this answer we have decided to keep the Moon's distance from the Earth at 12012.5km, which is 1/32 of the Moon's current distance away from the Earth. This distance is well above the Roche limit and is "safe" to use.

The tidal forces due to the Moon causes the acceleration of water away from the Earth, however, the water on the other side of the Earth accelerates due to the centrifugal force. To determine if the heights of the tides on both sides are the same we need to calculate the tidal acceleration on both sides of the Earth and compare the two. The equation for the tidal acceleration on the far side (away from Moon), a_F:

$$a_F = -\left(2\frac{GmR}{r_L^3} - 3\frac{GmR^2}{r_L^4}\right) \qquad (2)[2]$$

Where G is the gravitational constant, R is the radius of the Earth; r_L is the distance between the centre of the Moon and to the centre of the Earth

The equation for the tidal acceleration on the near side a_N:

$$a_N = \left(2\frac{GmR}{r_L^3} + 3\frac{GmR^2}{r_L^4}\right) \qquad (3)[2]$$

Using the equations we got $a_F = 1.07 \times 10^{-6}\text{ms}^{-2}$, and $a_N = 1.13 \times 10^{-6}\text{ms}^{-2}$, due to the small difference in the results we have assumed that the heights of these tidal bulges are the same.

$$h = \frac{3m}{2M}\left(\frac{R}{r_L}\right)^3 R \qquad (4)[3]$$

Equation 4 gives us a simplistic mathematical calculation for the height (h) related to the Moon-Earth distances. This equation assumes that the Earth is a non-rotational body and ignores the tidal force effects on the continents.
Figure 2 depicts the results of equation 4, and shows use the tide height for different Moon-Earth distances, the vertical line indicates the Roche limit. From the graph it can be seen that as the distance between Earth and Moon decreases the tidal heights increase.

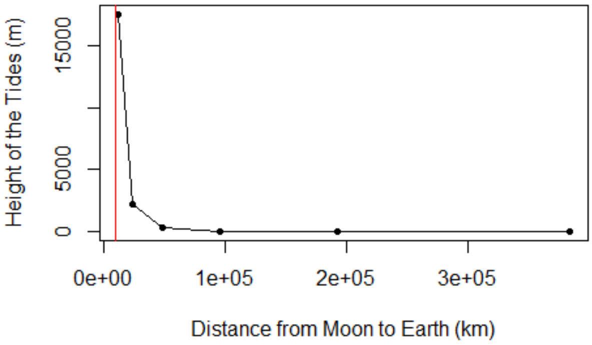

Figure 2 – A graph representing the relationship between the tide height and the distance between the Earth and the Moon

Conclusion:

Equation 4 is a mathematical representation of the tide with many constraints; realistically there are many factors that would contribute to the height of the tides such as; the coastline and the nearby continental shelf. The results we have are an extreme version of what would happen. If the Moon were brought to these close distances the planet, we could expect global floods due to the tidal waves reaching height of several kilometres, this would dramatically affect the coastlines. Shortening of the Moon's distance doesn't just affect the tides on Earth, the planet as a whole will experience changes such as; its rotation and the shape, this can be further investigated.

Reference:

[1] F. H. Shu, *The Physical Universe: an Introduction to Astronomy*, (University Science Books, 1982), p. 431.
[2] W. Lowrie, *A Student's Guide to Geophysical Equations,* (Cambridge University Press, 2011), p. 118.
[3] https://squishtheory.wordpress.com/the-tides/ accessed on 28/10/2015.