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A2_5 A Zombie Epidemic

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Abstract

We investigate the spread of a zombie virus through the global population with one person infected at day 0, using the SIR model. We find that by day 100 the surviving population is roughly 100-200 people.

Introduction

The SIR model is an epidemiological model that describes the spread of a disease throughout a population. This model splits the population into three categories: Those susceptible to the infection, those that are infected, and those that have either died or recovered. The SIR model then considers the rates at which infections spread and die off as individuals in the population come into contact with each other. We will use this model to investigate the spread of a zombie epidemic.

The basic SIR model

The SIR Model is defined by the following three ordinary differential equations [1]

$$\frac{dS}{dt} = -\beta \frac{SZ}{N} \quad (1)$$

$$\frac{dZ}{dt} = \beta \frac{SZ}{N} - \gamma Z \quad (2)$$

$$\frac{dD}{dt} = \gamma Z \quad (3)$$

where S is the susceptible population, Z is the zombie population and D is the dead population. $\frac{SZ}{N}$ is the availability for an encounter between a susceptible human and a zombie. β gives the probability of infection upon an encounter. The constraint upon these encounters is that a person always moves from $S \rightarrow Z \rightarrow D$, not $S \rightarrow D$. γ represents the reciprocal of the average lifetime of a zombie (the time constant of a zombie). N is the total population being investigated. The time frame over which individuals in the population encounter each other is given by the increment dt. If dt represents one day, then each day, $\beta\frac{SZ}{N}$ people will become infected and γZ zombies will die, moving them through the respective population groups.

We set the parameters as $\beta = 0.9$ (roughly twice as contagious as black death [2]) and $\gamma = 0.05$, for a zombie lifetime of 20 days before starvation and thirst renders it effectively dead [3]. We also took the population as $N = 7.5 \times 10^9$.

From epidemic to pandemic

We now alter the model to consider the geographical separation of sub-populations.

$$\frac{dS_i}{dt} = -\beta \frac{S_i Z_i}{N_i} \quad (4)$$

$$\frac{dZ_i}{dt} = \beta \frac{S_i Z_i}{N_i} - \gamma Z_i + \eta Z_{i\pm1} \quad (5)$$

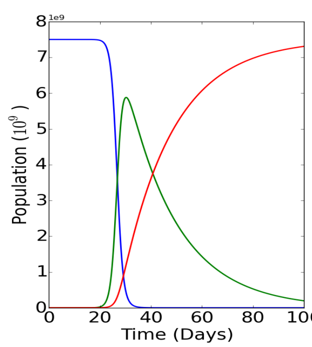
$$\frac{dD_i}{dt} = \gamma Z_i \quad (6)$$

where the population is split into i adjacent geographically isolated regions, each with their own S,Z and D sub-population types and total sub-population N_i (e.g. 2.5×10^9 for $i = 3$) such that the global population remains unchanged.

An additional term has been added in Eq (5) which accounts for the rate at which zombies are able to cross the geographical barriers into adjacent territory and spread the infection further. Where $i \pm 1$ are the adjacent regions to the i^{th} region. As such an infection starting at $i = 1$ will spread from regions $1 \rightarrow 2 \rightarrow 3$, but not $1 \rightarrow 3$ in the case of 3 geographically isolated regions.

The rate at which zombies can cross the geographical barriers is given by η. We have set $\eta = 10^{-5}$ so that the zombies are only able to leave their region once there are 10^5 zombies in their region. As such, fractions of zombies are rounded down. This prevents, for example, 0.01 zombies starting an infection across a geographical barrier when the population in their region is 10^3 and hence below the permitted threshold to advance the infection into the next region.

Results



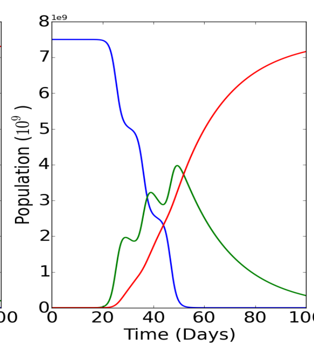


Figure 1: (Left) The spread of the zombie virus given by the curves S (blue), Z (green) and D (red), with initial conditions $S = N$, $Z = 1$, $D = 0$ using the basic model.

Figure 2: (Right) The spread of the zombie epidemic through 3 geographically isolated regions with initial conditions $S_i = N/3$, $Z_1 = 1$, $Z_{i>1} = 0$ $D_i = 0$.

Fig (1) shows that for the basic model, with only one person infected at day 0, it takes 20 days for the infection to spread to a noticeable fraction of the population. From this point the remaining population is quickly wiped out. By day 100 we find that there are 181 survivors and 1.9×10^8 zombies.

Fig (2) shows the global population change as the zombie infection evolves for 3 separated regions, assuming the global populations are evenly distributed and the zombie infection starts at $i = 1$. The spread of infection is staggered due to the geographical segregation, leading to three peaks in Fig (2) as the infection peaks in each region. The new model leaves 273 survivors at day 100, and again roughly 10^6 as many zombies.

Discussion

Natural birth and death rates have been neglected since the epidemic takes place over 100 days, so the natural births and deaths are negligible compared to the impact of the zombie virus over the short time frame. We also assumed that a zombie will turn one person into a zombie each day with a 90% probability, meaning that each zombie is able to find a person every day. As the zombie to human ratio increases, this becomes less realistic. A more developed model might vary β. We have also not included the possibility for the humans to kill the zombies. Including this may give the humans a better chance at survival.

References

[1] https://services.math.duke.edu/education/ccp/materials/diffcalc/sir/sir2.html accessed on 18/10/2016

[2] https://en.wikipedia.org/wiki/Black_Death accessed on 31/10/16

[3] http://www.livestrong.com/article/523013-how-many-days-can-you-survive-without-food/ accessed on 31/10/16